1. Consider the 100x100 symmetric probability transition matrix A from "transition_matrix_A.txt".

   (A) Argue what is the equilibrium distribution for this probability transition matrix.

   (B) Compute the first left eigenvector of matrix A (corresponding to the largest eigenvalue). Compare this eigenvector to the equilibrium distribution. (you can use publicly available packages for computation of eigenvalues and eigenvectors)

   (C) Simulate a Markov chain using the above matrix A by
      a. Sampling the first state X_0 (e.g. based on uniform distribution)
      b. Jumping from state X_i to state X_{i+1} according to the transition probability defined by *A.*

   For M=10000, what is the portion of time Markov chain spends in each of the 100 states ? Compare the this distribution to the expected equilibrium distribution using Euclidian distance as a metric. How fast the distribution approaches the expected distribution as M increases ?

2. (A) Use the Metropolis-Hasting approach to MCMC, for introducing rejection probabilities in the probability transition matrix such that the equilibrium probability distribution is equal to $P_{desired}$ from "desired_equilibrium_distribution.txt". In Metropolis-Hasting approach, the rejection probabilities are defined as :

$$R_{ij} = min(1, p_j.A_{ij}/ p_i.A_{ji})$$

   (B) Compute the probability transition matrix B after applying the rejection probabilities.

   (C) Compute the first left eigenvector of matrix A (corresponding to the largest eigenvalue).Compare this eigenvector to the desired equilibrium distribution.

   (D) Simulate a Markov chain using matrix B and approximate equilibrium distribution after M=10000 samples. Compare this equilibrium distribution to the desired equilibrium distribution (using Euclidean distance metric).