# Mobility Modeling, Spatial Traffic Distribution, and Probability of Connectivity for Sparse and Dense Vehicular Ad Hoc Networks

G. Hosein Mohimani, Farid Ashtiani, *Member, IEEE*, Adel Javanmard, and Maziyar Hamdi

***Abstract*—The mobility pattern of users is one of the distinct features of vehicular ad hoc networks (VANETs) compared with other types of mobile ad hoc networks (MANETs). This is due to the higher speed and the roadmap-restricted movement of vehicles. In this paper, we propose a new analytical mobility model for VANETs based on product-form queueing networks. In this model, we map the topology of the streets and the behavior of vehicles at both intersections and different parts of the streets onto different parameters of a BCMP[1] open queueing network comprising M/G/$\infty$ nodes. This model represents a sparse situation for VANETs. To include the effect of dense situation on the mobility model, we modify the proposed queueing network as a new one comprising nodes with state-dependent service rates, i.e., M/G$(\mathrm{n})/\infty$ nodes. With respect to the product-form solution property of the proposed queueing networks, we are able to find the spatial traffic distribution for vehicles at both sparse and dense situations. Furthermore, we are able to modify the proposed queueing network to find the lower and upper bounds for the probability of connectivity. In the last part of this paper, we show the flexibility of the proposed model by several numerical examples and confirm our modeling approach by simulation.**

***Index Terms*—Connectivity, mobility model, queueing network, spatial traffic distribution, vehicular ad hoc network (VANET).**

## I. Introduction

The MOBILE ad hoc network (MANET) is one of the hot research fields in communications. Up until now, several types of MANETs, e.g., vehicular ad hoc networks (VANETs), have been considered in the literature. Because of advancements in different technologies, popularity of some new products (e.g., digital maps, GPS, etc.), and the importance of safe transportation, VANETs have attracted much attention nowadays. Two basic types of services have been considered for VANETs, i.e., emergency services for safety applications and conventional communication services. The latter is important to make VANETs commercially beneficial.

VANETs have special attributes that differentiate them from the other types of MANETs. One of the main distinguishing features reverts to the behavior of their users. For a typical VANET, the mobility of users is a key factor in designing different aspects of the network, including Media Access Control (MAC) and routing protocols. Succinctly, users in VANETs are faster than users in conventional MANETs. Moreover, the mobility patterns of users in a VANET are more restrictive, hence more predictable than those of in a conventional MANET. These two characteristics, i.e., higher speed of users and more restricted mobility patterns, are very effective in most of the previously proposed MAC and routing protocols. Therefore, suitable means for modeling the mobility patterns, i.e., mobility models, play a key role in the performance evaluation of VANETs.

Up until now, several mobility models have been proposed in the literature. Some of them are suitable for simulation [1]–[3]. Some of the others have been proposed for cellular networks (particularly [4]). A few of them are suitable for both simulation and analysis [4]–[6]. Moreover, some mobility models have focused on the statistical parameters of the users, e.g., sojourn time at each region [7]. Although a few of the earlier models, e.g., random waypoint model [6], have been proposed and employed in MANETs [8], including the topology of the streets and considering both sparse and dense situations are not appearing to be easily applicable. In sparse situations, the mobility of the vehicles is considered independent from each other; however, for dense situations, such an assumption is not correct. In fact, in VANETs, the physical dimensions of vehicles are remarkable when compared with the width of the streets. This characteristic of users in VANETs (i.e., vehicles), in general, leads to the dependency of the mobility pattern of a vehicle on the other ones. To the best of our knowledge, a suitable analytical mobility model for dense situations has not been proposed in the literature.

In this paper, we propose a mobility model capable of including the behavior of vehicles in sparse and dense scenarios. To this end, we employ a BCMP open queueing network comprised of nodes with state-independent and state-dependent service times, respectively [9]. The proposed queueing network is suitable to include the topology of the streets. In addition, we are able to include asymmetric topologies, i.e., streets with different widths. With respect to the product-form solution property of the proposed queueing network, we are able to find

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[1]Baskett, Chandy, Muntz, and Palacios (BCMP).

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the spatial traffic distribution for vehicles at different streets. Spatial traffic distribution is a very important topic for designing and evaluating suitable routing algorithms in VANETs. The mobility model proposed in [4] has also been based on queueing networks; however, it is not suitable for VANETs because the road topology and dense situations have not been considered. It is worth mentioning that the proposed model in this paper focuses on the steady-state behavior of the vehicles. In fact, the behavior of the vehicles is a time-varying process. Then, vehicles behave differently in different time intervals, e.g., in the morning and evening; however, it is assumed that at each time interval their behavior is approximately stationary and predictable. In addition, it is assumed that each time interval is sufficiently large such that the behavior of the vehicles reaches its steady state. Thus, we are able to apply the proposed models on each time interval by setting the parameters in a suitable manner, e.g., by statistical data gathering at similar time intervals.

One of the other important problems in VANETs is the probability of connectivity. Several definitions have been considered for this probability in the literature. Although a few approaches have been proposed in the literature for computing the probability of connectivity in MANETs [10]–[13], most of them are only applicable to 1-D cases. The probability of connectivity in VANETs has two delicate features; it is a 2-D problem, and the road topology deeply affects this probability. The probability of connectivity has been considered for VANETs in different views [5], [13]–[16]. In this paper, we consider a general definition for connectivity and offer a suitable modification on our proposed mobility model to find lower and upper bounds for the probability of connectivity along a street. Our approach is able to investigate the effect of the different parameters of the mobility pattern and the radio transmission range of vehicles on the bounds for the probability of connectivity.

Following this introduction, in Section II, we discuss the general characteristics and some assumptions for the mobility patterns of vehicles. In Section III, we propose a queueing network as the mobility model with sufficient capabilities to represent the mobility patterns of vehicles. Then, we modify the proposed queueing network to make it suitable for dense and sparse scenarios. We also obtain the spatial traffic distributions for sparse and dense situations in Section III. Section IV deals with the connectivity problem and shows the flexibility of the proposed mobility model in computing lower and upper bounds for the probability of connectivity. In Section V, we show the flexibility of the proposed model by several numerical results and confirm the results by simulation. We conclude this paper in Section VI.

## II. General Characteristics of Mobility Patterns

As discussed in the previous section, the mobility pattern of each vehicle is strongly affected by the topology of the roads. Then, to consider a mobility pattern, we focus on a desired region such as in Fig. 1. However, there is no restriction on the shape and direction of the streets. In the desired topology (Fig. 1), we have several intersections and streets. We assume that the vehicles arrive in the desired region from entrances

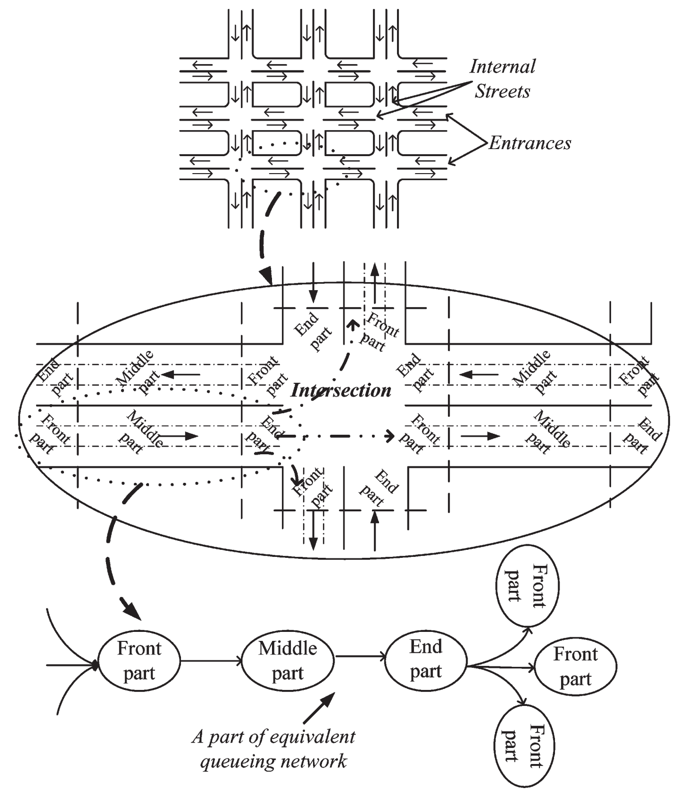


Fig. 1. Typical roadmap (road topology) considered in this paper and the method of mapping different parts of the streets onto queueing nodes.

according to a Poisson process. Since the behaviors of vehicles along the streets are usually different, we differentiate the movement of a vehicle along different parts of a typical street. In this respect, we assume that a typical vehicle at each street has three phases of movement. For the sake of simplicity, we only focus on the average speed of the vehicles at each phase of movement as in the following.

The first phase of the movement denotes a starting phase that corresponds to the front part of the street. In this phase, each vehicle has a varying speed until it reaches a nearly constant speed. The vehicle maintains its speed at the middle part of the street, equivalent to the second phase of its movement. This constant speed depends on the driving habits of the driver and other factors in the street. By considering vehicles at that street in a statistical sense, we have assumed three categories of speeds for the middle part, i.e., low, medium, and fast. In addition, for vehicles parking at that street (only at the middle part), we have considered a forth category of speed, i.e., very low. Each category at the preceding division has a range of speeds as well as a general distribution (e.g., uniform), such that each vehicle of a typical category selects a speed from the corresponding range according to the corresponding distribution. After moving along the middle part of the street and approaching an intersection, we have considered another phase of movement, i.e., ending phase. We have considered two categories of speeds, in general, for this end part of the streets, i.e., low and medium. Low speeds represent the situation of encountering stoplights, stop signs, caution for turning left or right, or any other means of hindering. Moreover, the medium speed category corresponds to the case of not encountering an effective obstacle. Correspondingly, we have considered two speed categories for the vehicles arriving in the front part, one represents the vehicles arriving at low speeds and the other one represents the vehicles arriving at medium speeds. Therefore, we have two speed categories at both front and end parts of each street and four speed categories for the vehicles at the middle part of the streets. For the sake of simplicity, it is assumed that the vehicles may change their speed category only at the end of each part. Moreover, we consider the length of the front and end parts of the streets very small compared to the length of the middle parts. On the other hand, we consider three lanes for each street (see Fig. 1). Each lane corresponds to a speed category. For example, in the middle part, the medium-, low-, and high-speed categories correspond to the second, first, and third lanes, respectively.

Another important feature of the mobility pattern is how the vehicles change their directions at intersections. A vehicle at an intersection can go straight forward, turn to the right, or turn to the left. For the sake of simplicity, it is assumed that vehicles at the second lane of the end part of each street go straight forward, and vehicles at the first and third lanes of the end parts turn to the right and left streets, respectively (see Fig. 1). Then, the vehicles at the first and third lanes of the end parts need to be more careful and reduce their speeds before turning. Therefore, we have assumed that the first and third lanes of the end parts correspond to the low-speed category, and the second lane corresponds to the medium-speed category. Similar assumptions are applied on the front part of each street. It is worth mentioning that these assumptions are only for the sake of simplicity and can easily be modified according to the driving laws at each city.

In the previously discussed mobility pattern, if the density of the vehicles at a typical street is low, i.e., a sparse scenario, we can consider the mobility of the vehicles independent of each other. However, if the density of the vehicles increases, the mobility patterns of the vehicles affect each other. In this case, the vehicles are obliged to change their lanes along a street to overtake each other or decrease their speed to avoid any crash. It is worth noting that we may have dense scenarios at some streets and sparse scenarios for the others. Moreover, the threshold corresponding to the number of vehicles that distinguishes dense and sparse scenarios is different for dissimilar streets.

In the next section, we discuss how to include the interdependency among vehicles for dense scenarios in the mobility model. It is worth mentioning that most of the previously proposed mobility models have not considered the dense scenarios. However, because of the remarkable dimensions of the vehicles compared with the width of the streets, topology of the streets, etc., dense scenarios are very common in VANETs. In the next section, we propose models capable of including the foregoing characteristics of the mobility patterns.

## III. Proposed Mobility Model and Computation of Spatial Traffic Distribution

In this section, we propose two mobility models for dense and sparse scenarios. In this respect, we consider a queueing network comprising several nodes and customers with several classes. In our modeling approach, the vehicles are mapped onto the customers of the queueing network. In addition, three nodes are considered for each street at each direction, which correspond to the front, end, and middle parts of that street (Fig. 1). Then, we correspond the sojourn time of a vehicle at each part of the street to the service time of the corresponding customer at the corresponding node. Therefore, in the queueing network, when a customer departs from a node representing the front or middle part of a typical street, it is definitely routed to a specific node, i.e., the node representing the middle or end part of the same street, respectively. However, when a customer departs the node representing the end part of a street, i.e., the corresponding vehicle reaches an intersection, it is routed to one of the three nodes representing the front parts of the other three streets at that intersection with different probabilities (Fig. 1).

According to the details of the vehicles' mobility patterns discussed in Section II, we consider different speed categories at different parts of the streets, approximately representing the different behavior of the vehicles. In the proposed mobility model, we simply map each speed category onto a customer class. Therefore, we have four customer classes at nodes representing the middle parts of the streets (including parking vehicles) and two customer classes for the nodes representing the front and end parts of the streets. Each customer class at each node has a random service time with a distribution derived from the distribution of the corresponding speed category at the corresponding street. Then, the average service time (equal to the inverse of the service rate) of a customer with a specific class at each node is determined with respect to the distribution of its corresponding speed and the length of street parts. In the following, we distinguish between sparse and dense scenarios in describing the other characteristics of the proposed queueing network.

### A. Sparse Scenario

As discussed in Section II, when a street is considered in a sparse scenario, we can consider different vehicles having independent mobility patterns. Therefore, we consider each node at the preceding queueing network as an M/G/$\infty$ node. So, in the case where all parts of the street are in a sparse situation, we actually have a queueing network comprising M/G/$\infty$ nodes with four, two, and two customer classes, corresponding to the middle, front, and end parts, respectively. This is a BCMP queueing network, i.e., a quasi-reversible queueing network with a product-form solution [9]. In this network, the following traffic equations are satisfied [9]:

$$\alpha_{ju} = \lambda_{ju} + \sum_{k=1}^{N} \sum_{v=1}^{C(k)} \alpha_{kv} r_{kv,ju} \qquad j = 1, \ldots, N, \qquad u = 1, \ldots, C(j) \quad (1)$$

where $\lambda_{ju}$ is the exogenous arrival rate of class-$u$ customers at node $j$, $N$ is the number of nodes (street parts), $C(j)$ is the number of classes at node $j$, and $r_{kv,ju}$ denotes the routing probability of a departed class-$v$ customer from node $k$ to node $j$ as a class-$u$ customer. $\lambda_{ju}$ corresponds to the arrival rate of vehicles into entrances of the desired region (Fig. 1), $C(j)$ corresponds to different speed categories, and $r_{kv,ju}$ depends on the behavior of the vehicles at the end of street parts and at intersections. In addition, $\alpha_{ju}$ in (1) denotes the arrival rate of class-$u$ customers at node $j$ in the network, which represents an exogenous arrival rate as well as the arrivals routed from other nodes. It obviously is equal to the departure rate of class-$u$ customers from node $j$ in the case of stability. We also have the following relation for routing probabilities [9]:

$$\sum_{k=1}^{N}\sum_{v=1}^{I} r_{ju,kv} + r_{ju,0} = 1; \quad j = 1,\ldots,N, \quad u = 1,\ldots,I \tag{2}$$

where 0 denotes the world outside the network (i.e., out of the desired region). The routing probabilities are determined with respect to the mobility patterns of the vehicles, as discussed in Section II. Then, we obtain the spatial traffic distribution for the proposed queueing network comprising M/G/$\infty$ nodes as in the following [9]:

$$\begin{aligned} \pi(\mathbf{n}) &= \prod_{j=1}^{N} \pi_j(\mathbf{n}_j) \\ &\qquad \mathbf{n} = (\mathbf{n}_1,\ldots,\mathbf{n}_N), \quad \mathbf{n}_j = \left(n_{j1},\ldots,n_{jC(j)}\right) \\ \pi_j(\mathbf{n}_j) &= e^{-\rho_j} \prod_{u=1}^{C(j)} \frac{\rho_{ju}^{n_{ju}}}{n_{ju}!} \\ &\qquad \rho_{ju} = \frac{\alpha_{ju}}{\mu_{ju}}, \quad \rho_j = \sum_{u=1}^{C(j)} \rho_{ju} \end{aligned} \tag{3}$$

where $\mu_{ju}$ denotes the service rate of a class-$u$ customer at node $j$, $N$ is the number of nodes (street parts), and $C(j)$ denotes the number of classes at node $j$. In (3), $\pi(\mathbf{n})$ represents the probability of $n_{11}$ class-1 customers at node 1, $n_{12}$ class-2 customers at node 1, etc. By (3), we obtain the stationary probability for different numbers of customers of different speed categories in different parts of the streets, i.e., spatial traffic distribution. Note that if we divide each street into parts with a smaller length, we will have the spatial traffic distribution with more details. In Section IV, we apply this modification to compute bounds for the probability of connectivity. In this case, we will have more nodes in the queueing network, leading to more computational complexity.

We may also be interested in knowing the stationary probability for different numbers of vehicles at a specific street when we have a fixed number of vehicles in the whole region. Then, we have, in fact, a closed queueing network. In this case, we can assume that when a vehicle goes out of the desired region, another vehicle symmetrically arrives in the desired region from the other side (i.e., wraparound technique). Then, we need to modify traffic equations to obtain the stationary distribution similar to (3), except for a normalizing constant [4], [9].

### B. Dense Scenario

As indicated in Section II, in dense scenarios, some of the vehicles cannot maintain their speed, irrespective of the vehicles in front of them. Then, the sojourn times of some vehicles at a street are dependent on the number of other vehicles at the same street. For more clarification, suppose a vehicle selects a speed independently and uniformly in the range $[v_{\min}, v_{\max}]$ with respect to a speed category. For a sparse scenario, since the vehicles independently move along the street, the average sojourn time at the street is equal to $(L/(v_{\max} - v_{\min}))\ln(v_{\max}/v_{\min})$, where $L$ is the length of the street. If the number of vehicles in that street increases (due to larger arrival rate), some of the vehicles are obliged to temporarily reduce their speed because they encounter some slower vehicles in front. In fact, when the street is crowded enough, there is no possibility for maneuvers and overtaking. Therefore, we can say, in general, that the distribution of speeds at that street deviates from uniform and diverts to a new distribution with more weight on lower speeds. This leads to a higher average sojourn time at that street. As long as the number of vehicles in a typical street increases, more vehicles during their movement along the street encounter slower vehicles such that they are obliged to reduce their speeds. Then, the sojourn time of more vehicles is affected by other vehicles in that street (i.e., slower vehicles at their fronts). This situation leads to a more observable deviation in the distribution of the number of vehicles at that street. Obviously, when the width of a street is smaller, the dense situation is observable for a smaller number of vehicles.

With respect to the preceding discussions, a street in a dense situation cannot be mapped onto an M/G/$\infty$ node. In other words, when a part of a street is considered in a dense situation, we need to include the effect of the number of customers on the service rate at the corresponding node. To include the foregoing dependency, we employ a queueing network comprising nodes with state-dependent service times, e.g., M/G(n)/$\infty$ nodes. Then, we modify the previously proposed BCMP queueing network and exploit the property of the functional product form, as explained in [9]. In the newly proposed queueing network, only the service rate at each node will be changed. Then, the traffic equations (1) will not be changed. However, the stationary probability distribution for traffic states, i.e., (3), is modified as

$$P(\mathbf{n}) = b_n \Phi(\mathbf{n})\pi(\mathbf{n}), \qquad \pi(\mathbf{n}) = \prod_{j=1}^{N} \pi_j(\mathbf{n}_j) \tag{4}$$

where $P$ denotes the stationary probability distribution of the modified (state-dependent) network, and $\Phi$ is a function representing the dependency of service rate on the traffic state, i.e., $\mathbf{n}$. In addition, $\pi$ is the stationary probability distribution of the corresponding state-independent queueing network, i.e., the same traffic state in the case of sparse scenarios, and $b_n$ is a normalizing constant. In this modified queueing network, the exogenous arrival rates and the service rates need to be modified as [9]

$$\lambda_{ju}(\mathbf{n}) = \lambda_{ju}\frac{\Psi(\mathbf{n})}{\Phi(\mathbf{n})}, \quad \mu_{ju}(\mathbf{n}) = \mu_{ju}\frac{\Phi(\mathbf{n} - \mathbf{e}_{ju})}{\Phi(\mathbf{n})} \tag{5}$$

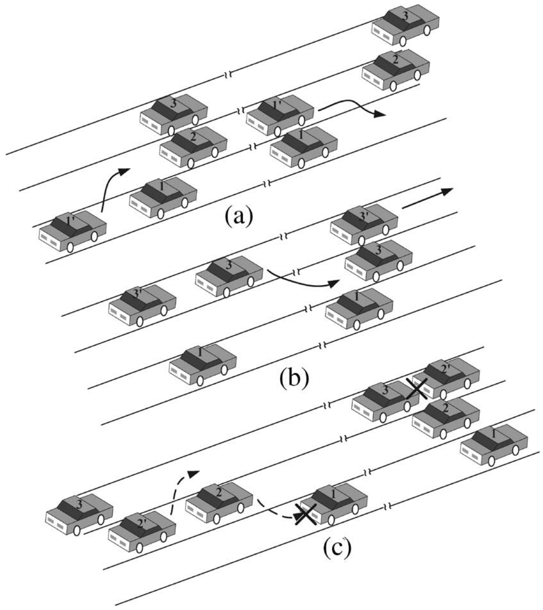


Fig. 2. Effect of sparse and dense situations on mobility pattern. (a) Possibility of overtaking by a simple faster vehicle's maneuver to the left. (b) Possibility of overtaking by a simple slower vehicle's maneuver to the right. (c) Impossibility of overtaking and any maneuver.

where $\mathbf{e}_{ju}$ represents a $(\sum_{j=1}^{N} C(j))$-dimensional vector with a 1 at position $ju$ corresponding to a class-$u$ customer at node $j$, and zero elsewhere. In general, $\Psi$ and $\Phi$ are arbitrary nonnegative and positive functions, respectively. In the proposed queueing network, we consider $\Psi$ equal to $\Phi$. Then, we obtain the same $\lambda$ as in the case of a sparse scenario. Now, we should determine $\Phi(\mathbf{n})$. To this end, we set $\mu_{ju}$ in (5) equal to 1 and consider a general form for $\Phi$ as

$$\begin{aligned}\Phi(\mathbf{n}) = \frac{1}{\prod_{j=1}^{N} \prod_{u=1}^{C(j)} \prod_{i=1}^{n_{ju}} \mu_{ju}(i)} &\Rightarrow \mu_{ju}(\mathbf{n}) \\ &= \frac{\Phi(\mathbf{n}-\mathbf{e}_{ju})}{\Phi(\mathbf{n})} \\ &= \mu_{ju}(n_{ju}). \end{aligned} \tag{6}$$

Then, we have different service rates for the different numbers of customers at each class. To compute $\mu_{ju}(n_{ju})$, we should consider more details for the mobility patterns in dense situations.

We have considered regular and lawful drivers such that the vehicles at each speed category do their best to remain in the corresponding lanes. In other words, it is assumed that whenever a vehicle changes its lane to overtake a slower one, it returns to its initial lane after overtaking. Moreover, whenever a vehicle of a speed category encounters a slower vehicle of the same speed category, the faster vehicle overtakes the slower vehicle without any increase in its speed and by a simple maneuver to the left if possible [see Fig. 2(a)]. If that is not possible, the slower vehicle yields the way to the faster vehicle to pass (i.e., overtake) by a simple maneuver to the right [see Fig. 2(b)], and after overtaking, the deviated vehicles return to their initial lanes. In these cases, we do not encounter any disturbance in the speed distribution or the average sojourn time. However, if maneuvers to the left and right are impossible [see Fig. 2(c)], then the faster vehicle temporarily decreases its speed until the obstacles for overtaking are removed. Therefore, the average sojourn time of a vehicle at a specific lane is dependent upon both the number of vehicles at the same lane and the number of vehicles at the other lanes. In other words, the average sojourn time depends on the total number of vehicles at three lanes. Therefore, we modify (6) such that $\mu_{ju}(.)$ depends on $n_j$ instead of $n_{ju}$. To this end, instead of considering several speed categories for moving vehicles (excluding parking vehicles) at each street part, we only consider one speed category (i.e., one class), which is a mixture of different speed categories at that street part.

We apply the preceding modification for the customers at all parts of the streets; however, for the middle part, we distinguish between vehicles parking at the roadside and vehicles moving along the street. In fact, parking vehicles have no effect on the moving vehicles. So, we consider one class of customers for both front and end parts of the streets, i.e., moving vehicles, and two classes of customers for the middle parts of the streets, i.e., moving and parking vehicles. Hence, we apply (6) for our proposed queueing network with stationary probability distribution (5) such that $C(j)$ is equal to 1 for the nodes corresponding to the front and end parts of the streets and is equal to 2 for the nodes corresponding to the middle parts of the streets. Moreover, in the latter case, $\mu_{ju}(n_{ju})$ is equal to a constant $\mu_{ju}$ for class-$u$ customers corresponding to parking vehicles.

To compute $\mu_{ju}(n_{ju})$ for moving vehicles in the middle parts or $\mu_j(n_j)$ in the front and end parts, we need some statistical tests (e.g., by simulation). We can intuitively say that the average sojourn time is a constant when the number of vehicles is small and will increase when the number of vehicles increases because of the significant number of speed changes. In other words, when a significant number of vehicles encounter obstacles while overtaking, we will have an increase in the average sojourn time. At the other extreme, we have another constant for the average sojourn time, denoting that we approximately have a saturation condition. Reasonably, in a regularly designed road topology, we are very far from the saturation condition.

We run a simulation to obtain $\mu_j(n_j)$ for the different numbers of customers for a typical street. In the simulation, we wrap any departing vehicle to keep the number of vehicles constant, and we apply the previously discussed overtaking laws in the simulation. To make the simulation more practical, we consider the possibility of a maneuver to the left for overtaking when there is no vehicle at the left lane within a specific distance. This distance represents the driver's field of view. If a vehicle exists within this distance, the driver decides to overtake based on the location of that vehicle. It is worth mentioning that the vehicles at the left lanes are faster than the vehicles at the right lanes. In Fig. 3, we have sketched the flowchart for our simulation. The foregoing discussion about the possibility of maneuvers to the left is also applied for maneuvers to the right to yield the path to faster vehicles.

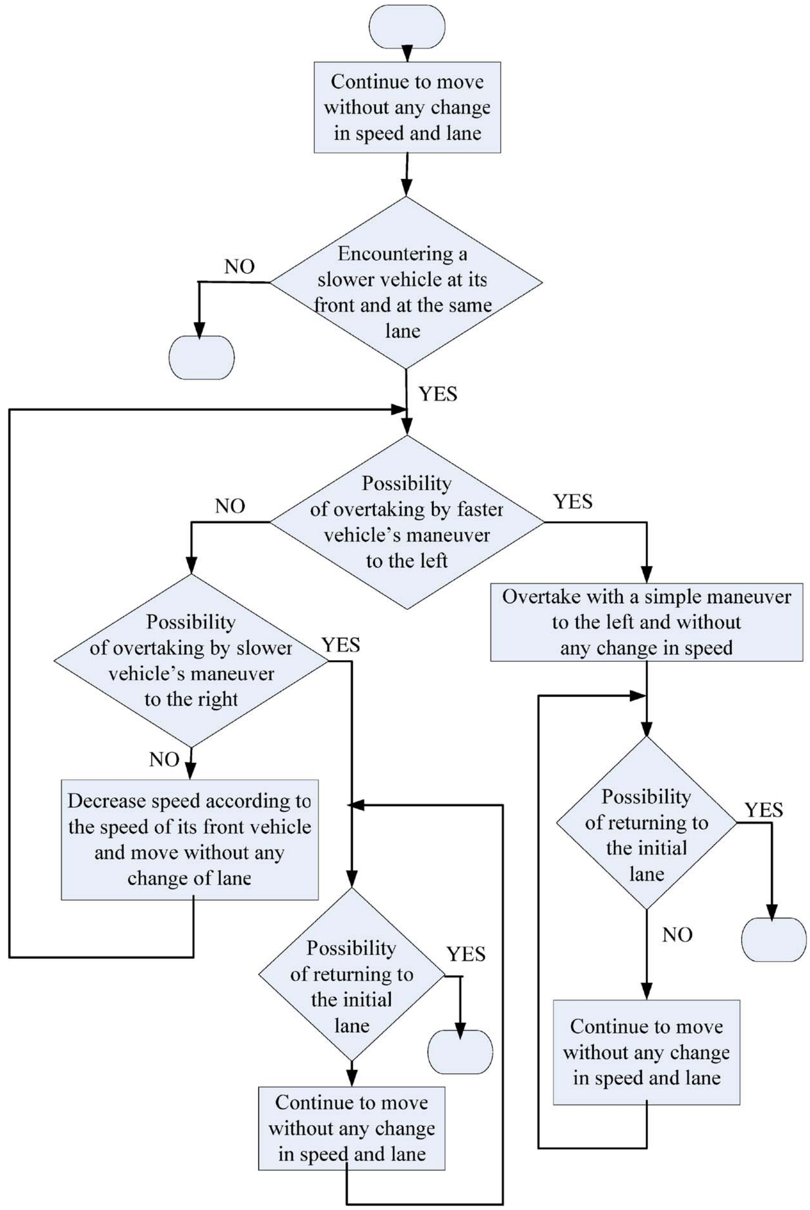


Fig. 3. Flowchart describing laws for overtaking and maneuvers applied in deriving Fig. 4 and in the simulations.

In Fig. 4, we illustrate the inverse of the average sojourn time for different numbers of vehicles at the middle part of a typical street with three lanes and the length of 1600 m by simulation. For example, when there are 500 vehicles in the middle part of the street, the departure rate of vehicles reduces to 80% of the rate corresponding to the sparse scenario (i.e., the case when there are a few vehicles in the street). We employ the results of Fig. 4 in (6), i.e., different service rates for different number of vehicles. By obtaining similar curves for all parts of the streets, we are able to obtain $P(\mathbf{n})$ in (4) for each traffic state $\mathbf{n}$.

## IV. Probability of Connectivity

As indicated in Section I, one of the important characteristics for a VANET is its probability of connectivity. The importance of connectivity reverts to designing efficient routing algorithms for VANETs. In a typical routing algorithm, the main goal is forwarding a packet from a source to its destination with minimum delay. This is very important for emergency services in a VANET. An interesting concept in the proposed routing algorithms for VANETs is the carry and forward approach [17]. In this approach, if a vehicle does not find a suitable vehicle to forward its packet, it carries the packet itself. Since packet forwarding through radio propagation is faster than packet carrying by a vehicle, the probability of connectivity is a key factor in designing efficient routing schemes. In fact, sometimes, routing the packets from the farther paths is more suitable than nearer paths because more connectivity at farther paths can reduce the delay of the received packets.

Several definitions for the probability of connectivity in a MANET have been proposed in the literature [10]–[16]. In the case of a VANET, the topology of the roads is very effective. Then, we consider the probability of connectivity as the probability that the vehicles along the streets can communicate with each other through a radio link. In general, it is possible to have several radio links between two vehicles. In addition, each radio

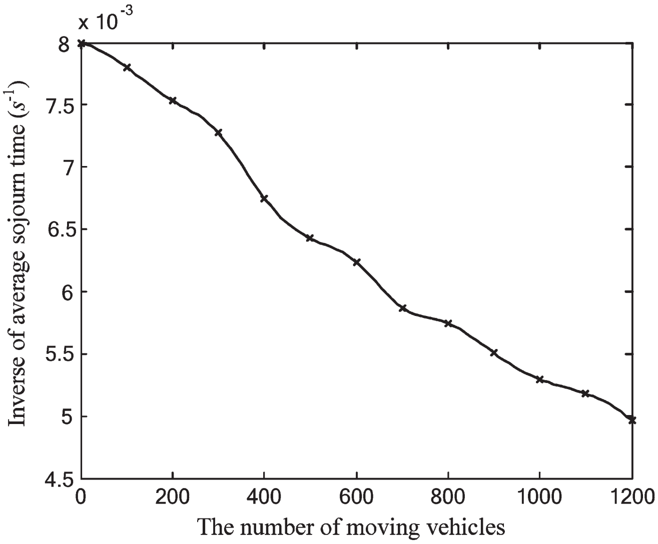


Fig. 4. Dependency of the inverse of the average sojourn time for moving vehicles on the number of vehicles in the middle part of a typical street.

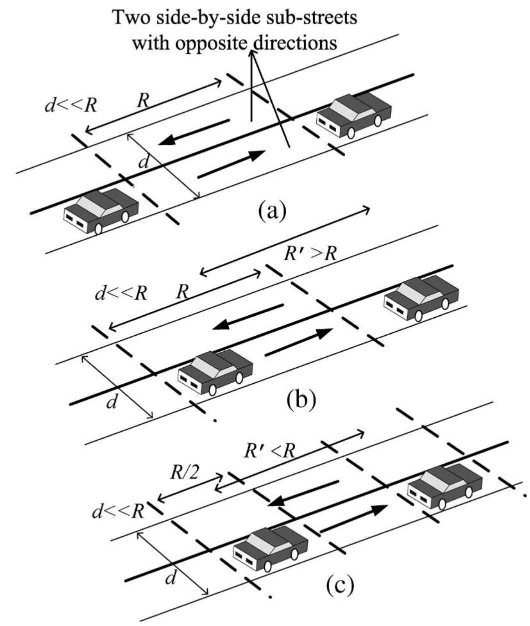


Fig. 5. (a) Certain nonconnectivity when any of the substreets with length $R$ is empty. (b) Possible nonconnectivity even if all substreets with length $R$ are nonempty. (c) Possible connectivity even if a substreet with length $R/2$ is empty.

link may consist of multiple hops. Two vehicles are able to directly communicate if they are in the radio transmission range of each other. Therefore, to find the probability of nonconnectivity, in general, we need to find the probability of interruption of all radio links between any two vehicles. It is worth noting that if a link is disconnected, e.g., along a street, we may have another radio link through other streets.

In this paper, we have focused on finding simple lower and upper bounds for the probability of connectivity along a typical street in the road topology as in Fig. 1. Obviously, in a street, we have a disconnected link if we have two consecutive vehicles with a distance greater than the radio transmission range $R$. It is worth mentioning that the direction and speed of vehicles are not important in this case. Therefore, we modify our proposed queueing network to find the probability of two consecutive vehicles at that street with a distance greater than $R$. To this end, we divide each part of the street (i.e., front, middle, and end parts) into substreets, each with length $R$. With respect to the previous approach in mapping parts of streets onto nodes in a queueing network, we will have a queueing network similar to the one proposed in Section III but with greater number of nodes. In this queueing network, a departed customer with a specific class from a node corresponding to a substreet is routed to the node corresponding to the following substreet as a customer with the same class. Moreover, if the substreets belong to different parts of the street, the customer class may change (due to the initial mobility pattern as discussed in Section II). Obviously, we will have traffic equations and stationary probability distribution similar to (1) and (3). To find the probability of nonconnectivity along a typical street, we find the probability that there is at least one pair of side-by-side empty substreets (nodes) with opposite directions [see Fig. 5(a)]. However, if we have such a pair of nodes, the connectivity does not necessarily hold along the street [e.g., see Fig. 5(b)]. Therefore, the computed probability is only a lower bound for nonconnectivity, which is equivalent to an upper bound for connectivity.

On the other hand, consider the case in which we divide the streets into substreets with length $R/2$. Then, if we have at least one vehicle at each substreet (irrespective of its direction and speed), we will have full connectivity for that street. However, if we have an empty pair of side-by-side substreets in this new structure, connectivity may also hold [see Fig. 5(c)]. In this case, connectivity depends on the locations of the vehicles in the following and preceding substreets. Therefore, by computing the probability with which all pairs of side-by-side nodes are nonempty, we will find a lower bound for connectivity.

With respect to the foregoing discussions, we have the following relations for the lower and upper bounds for the probability of connectivity at a typical street:

$$\begin{aligned} P(Connectivity) &\geq \sum_{\substack{n_i \geq 1 \\ \forall i, 1 \leq i \leq N_S(R/2)}} \pi\left(n_1, n_2, \ldots, n_{N_S(R/2)}\right) \\ &= \prod_{i=1}^{N_S(R/2)} \sum_{n_i \geq 1} \pi(n_i) \\ &= \prod_{i=1}^{N_S(R/2)} \left(1 - \pi(n_i = 0)\right) \end{aligned} \tag{7}$$

$$\begin{aligned} P(Connectivity) &\leq \sum_{\substack{n_i \geq 1 \\ \forall i, 1 \leq i \leq N_S(R)}} \pi\left(n_1, n_2, \ldots, n_{N_S(R)}\right) \\ &= \prod_{i=1}^{N_S(R)} \sum_{n_i \geq 1} \pi(n_i) \\ &= \prod_{i=1}^{N_S(R)} \left(1 - \pi(n_i = 0)\right) \end{aligned} \tag{8}$$

TABLE I
TYPICAL VALUES FOR THE PARAMETERS IN THE NUMERICAL ANALYSES

| *Parameter* | *Value* | *Unit* |
|---|---|---|
| Low speed category in the middle part | Uniform [3, 14] | *m/s* |
| Medium speed category in the middle part | Uniform [14, 22] | *m/s* |
| Fast speed category in the middle part | Uniform [22, 33] | *m/s* |
| Low speed category in the front part | Uniform [0.3, 3] | *m/s* |
| Medium speed category in the front part | Uniform [3, 14] | *m/s* |
| Low speed category in the end part | Uniform [0.3, 1.5] | *m/s* |
| Medium speed category in the end part | Uniform [1.5, 14] | *m/s* |
| Length of the middle part | 1600 | *m* |
| Length of the front part | 200 | *m* |
| Length of the end part | 200 | *m* |
| Arrival rate at each entrance (λ) | 0.8 | $s^{-1}$ |

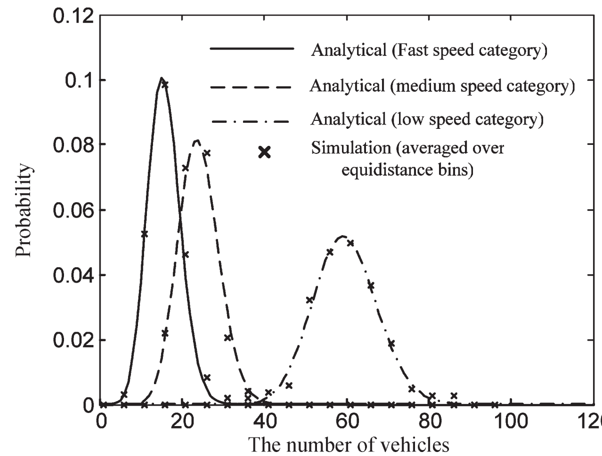


Fig. 6. Stationary probability distribution (state-independent model) for the number of vehicles with different speed categories in the middle part of a typical street in the case of uniform mobility pattern ($\lambda = 0.8$ vehicles/s).

where $N_S(R/2)$ and $N_S(R)$ denote the number of substreets for a typical street ($S$), where the length of each substreet equals $R$ and $R/2$, respectively. In the preceding relations, we have only focused on a typical street. It is worth mentioning that in writing (7) and (8), we exploit the product-form solution of the stationary probability distribution as in (3). In addition, in (7) and (8), we remove indexes representing different classes because only the existence of a vehicle (irrespective of its class and direction) is important. In fact, the probabilities in (7) and (8) will be obtained by applying (3) for sparse scenarios and (4) for dense scenarios and adding some related terms (e.g., addition of probabilities corresponding to two side-by-side substreets with opposite directions). It is worth noting that we consider the width of the streets very small compared to the transmission range $R$ in the preceding analytical approach.

Although the preceding analytical approach is considered along a typical street, it is applicable to intersections as well. However, we should consider four streets meeting the intersection and correspondingly modify the equations. It is worth mentioning that connectivity at intersections is of less importance than along the streets. The reason is that different hindering effects, e.g., stoplights, drivers' caution before turning, etc., lead to a higher density of vehicles at the intersections, i.e., more probability of connectivity.

## V. NUMERICAL RESULTS

In this section, we show the flexibility of the proposed model in computing the spatial traffic distribution for a VANET with a roadmap of Fig. 1. Table I indicates the typical values for some of the parameters related to the mobility patterns of the vehicles. Although in our numerical analysis we have assumed the same arrival rate at all entrances, we are able to consider cases that are more general. Moreover, for the sake of simplicity, we have ignored parking vehicles in our analyses. To confirm the numerical results, particularly for the case of a dense scenario, we simulate the mobility patterns of the vehicles in the roadmap of Fig. 1. The simulation is based on MATLAB, and the flowchart in Fig. 3 has been applied on all streets.

In the first set of numerical analyses, we have considered sparse scenarios and uniform mobility pattern. For a uniform mobility pattern, at each intersection, the vehicles go to one of three possible streets with equal probabilities. Fig. 6 illustrates the stationary probability for different numbers of customers with three speed categories at the middle part of a typical street in a sparse scenario. With respect to (3), the distribution is actually a Poisson distribution with parameter $\rho_{ju} = \alpha_{ju}/\mu_{ju}$. This parameter is obtained by solving traffic equation (1) and finding $\alpha_{ju}$. Moreover, $\mu_{ju}$ is determined with respect to the length of the middle part and the distribution of the corresponding speed category (see Section III). The simulation results are in good match with the analytical results. In addition, the simulation results confirm the Poisson distribution predicted by the model in a sparse scenario.

In Fig. 7, we have shown the effect of including a dense situation. In this respect, we employ (6) and the results obtained in Fig. 4. For comparison, we have plotted the stationary probability distribution for moving vehicles in the middle part of a typical street in the case of uniform mobility pattern for both sparse and dense situations. As we observe from Fig. 7, the probability of a higher number of vehicles (i.e., more crowds) increases in a dense situation, showing that if we do not consider the dependency among the mobility patterns of vehicles, we will have remarkable errors. Such a difference is noticeable when the arrival rate at the entrances increases. By running the simulation program for both sparse and dense scenarios, the validity of our modeling approach is confirmed. Moreover, as we observe from the mean and variance of the simulation results, the distribution in dense scenarios deviates from the Poisson distribution.

In Fig. 8, we have shown the efficiency of our proposed model in computing the stationary probability distribution for the case of an inward mobility pattern. In this case, the movement of vehicles at each intersection indicates that the vehicles are more interested in going toward the center

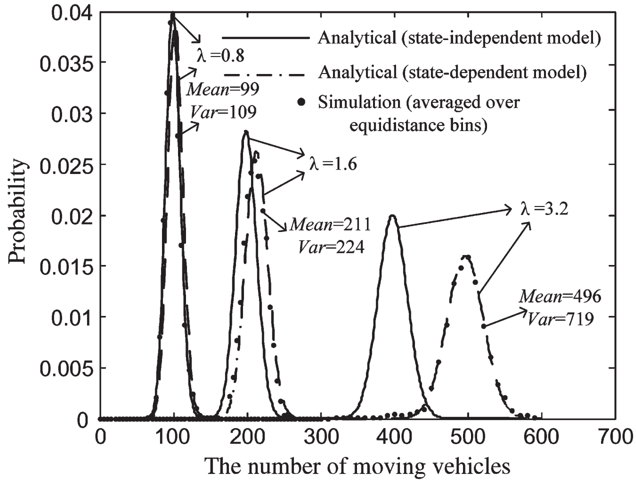


Fig. 7. Stationary probability distribution for the number of moving vehicles in the middle part of a typical street in the case of uniform mobility pattern with three arrival rates at each entrance with state-dependent service rates (4) and independent service rates (3).

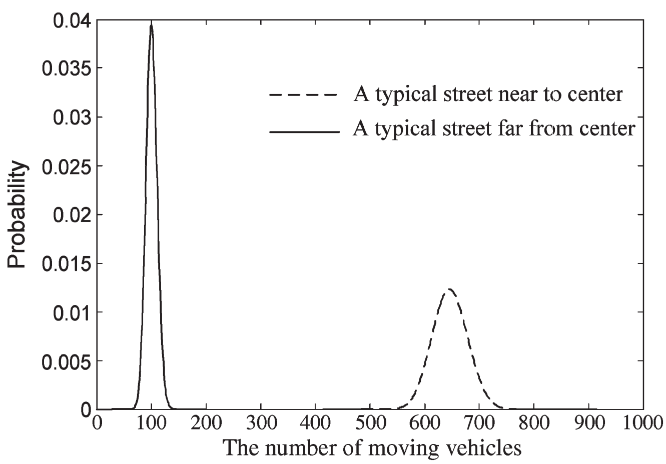


Fig. 8. Stationary probability distribution for the number of vehicles within medium speed category in the middle parts of two typical streets in the case of inward mobility pattern.

of the region. In fact, this case is an example of combined sparse and dense situations. In this figure, we compare the stationary distribution of vehicles in the middle parts of two streets, i.e., one near the center of the region and the other far from that. As expected, more vehicles concentrate on the central region, which leads to dense situations at the related streets. In obtaining the results of Fig. 8, we applied the state-dependent queueing network model. This figure shows the effect of decisions of the drivers on the stationary distribution at different streets.

Fig. 9 shows the lower and upper bounds for the probability of connectivity when the transmission range is 200 m. From this figure, it is obvious that we have a probability of connectivity that is more than 90% for the arrival rate nearly equal to 0.35 vehicles/s. With respect to (7) and (8), the lower bound for the case of $R = 400$ m and the upper bound for the case of $R = 100$ m have also been illustrated in Fig. 9. Since the probability of connectivity is of crucial importance in the case

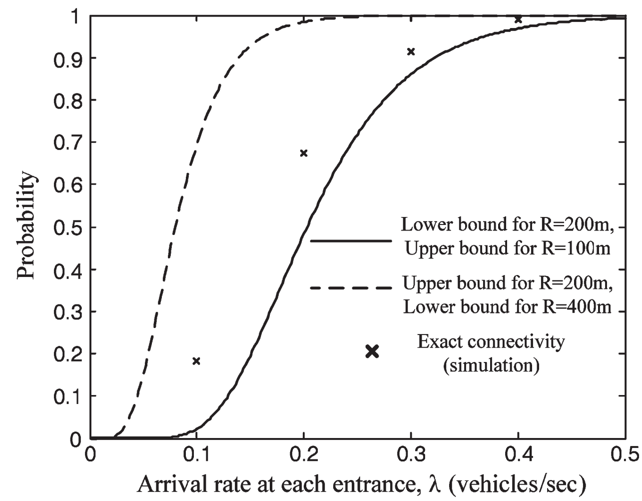


Fig. 9. Lower and upper bounds as well as exact value for probability of connectivity in a typical street in the case of uniform mobility pattern.

of a sparse scenario, we have not carried out a similar analysis for finding the bounds for the probability of connectivity in dense scenarios. In other words, the probability of connectivity reaches 1 for arrival rates smaller than the rates corresponding to dense scenarios (compare Figs. 6, 7, and 9). The simulation results for four arrival rates confirm the obtained analytical bounds.

Although the simple lower and upper bounds are not tight, however, the simple bounds are very suitable for finding the arrival rates at which different streets are nearly fully connected. Nevertheless, in general, they are not suitable as good estimates on the probability of connectivity.

## VI. Conclusion

Due to the high speed and roadmap-restricted movements of vehicles, mobility pattern is a key factor in VANET. We have employed a BCMP open queueing network to model the mobility patterns of vehicles in sparse and dense scenarios. To this end, we mapped different aspects of the mobility patterns, including different speed categories, the manner of decision making at intersections, and the distribution of speed onto different parameters of the proposed queueing network. To include the effect of dense situations, we modified the proposed BCMP queueing network as a new network comprising nodes with state-dependent service times. Then, we discussed the functional product form of such a network and determined the service rate at each node. Finally, we obtained the stationary probability distribution for different numbers of vehicles at different parts of the streets.

Furthermore, we focused on the probability of connectivity along a street and found lower and upper bounds for it. In this respect, we modified the proposed queueing network to find the bounds by employing the stationary probability distribution for some specific traffic states at those networks. Finally, we showed the flexibility of our modeling approach by some numerical examples and confirmed the validity of the proposed models by simulations.

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